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The Application of Operations Research to Industry

by

Ellis A. Johnson, Director

OPERATIONS RESEARCH OFFICE
The Johns Hopkins University Chevy Chase, Maryland

A Speech
Delivered at the
Fifth Annual Industrial Engineering Institute
University of California
January 31 and February 3, 1953

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THE APPLICATION OF OPERATIONS
RESEARCH TO INDUSTRY

INTRODUCTION

Operations research, or opsearch, as it is often abbreviated, is the new name for an old management technique which appears to be of increasing interest to industrial executives. Many of its older roots are held in common with industrial engineering, and its beginnings, like those of industrial engineering, go deep into antiquity. Its aboveground growth and application were slight and sporadic until World War II, when, as an aid to military management (especially in combat operations), it underwent a mushroom-like growth.

Today, in addition to the groups doing operations research for the Defense establishments, there are many business, scientific, and educational organizations interested in the field, and in many cases actively contributing to it. The Appendix, which includes organizations engaged in military as well as industrial operations research, gives an idea of the wide interest that now prevails. Further evidence of the growing interest is the organization of the Operations Research Society of America, a professional society of operations analysts, which now has a membership of about 500 and publishes a quarterly journal. It seems evident that operations research has become a profession in its own right.

There is some reason to believe that at the present time the methods of operations research are closer to those of the basic sciences than to those of engineering. The majority of operations research practitioners tend to come from the basic sciences rather than engineering; they use the attitudes and tools of physics rather than of electrical engineering, of psychology and sociology rather than of personnel management studies. However, the techniques and methodology of operations research do have a great deal in common with the techniques and methodology of industrial engineering and management consultants.

It is clear that operations research has been concerned from the start with the decision-making system and with the problem of providing individual executives with management advice. It has a role, therefore, similar to that of industrial engineering.

Like industrial engineering, it has, on behalf of the executive, been concerned with the development or calculation of future courses of action on the basis of exceedingly detailed knowledge gained from experience. It has differed from industrial engineering in its more conscious and more overt recognition of the need to get specialists, highly skilled in each field of knowledge having bearing on the particular action problem, to work in more closely integrated team research. Opsearch probably has been more systematic in its attempt to develop action models based on fundamental theory, similar to the modelmaking of the basic sciences, and in its heavy reliance on the more complicated mathematical concepts and techniques. It has, perhaps more than industrial engineering, been conscious of the need to estimate the uncertainties in its predictions when these have been concerned with tactical and strategic action problems, where the parameters are heavily dependent upon human behavior, upon assumptions with respect to competitor intentions, or upon imprecise intelligence knowledge of competitor capabilities.

I believe that a difference in timing is another of the distinguishing characteristics between the kinds of studies made by industrial engineers and management consultants, and those made by operations analysts. An operations research study usually takes a longer time to complete than a study by either one of its predecessors in the field, the tendency being to carry out studies of new courses of action which involve longer-range futures. These seem to be more in the nature of research studies than of engineering.

Operations research places a particular demand on the operations analyst's ability to translate his findings into executive language — that is, into language which simply and clearly sets forth the values, effectiveness, and costs of a set of proposed courses of action.

In the remaining sections of this paper I shall give chief attention to outlining methods of operations research, with the hope that this may assist in its further and broader application. I shall discuss four elements of operations research as follows:

I. The Relation of the Operations Analyst to the Executive
II. The Operations Research Checklist for Solving Action Problems
III. Some Selected Analytical Tools Used in Operations Research
IV. Simple Case Histories in Operations Research

I. THE RELATION OF THE OPERATIONS ANALYST TO THE EXECUTIVE

Let me remind you here of the nature of the decision-making problem faced by the executive. In Fig. 1, I have tried to show diagrammatically the principal activities controlled by the executive of a large organization—including his management, intelligence, and planning actions—although not, of course, in full detail. This is the usual kind of diagram showing the feed-back loops, or at least some of them.

The first point I would like to make is that applied research occurs at several points in this system. Three different kinds of research are shown in Fig. 1. The first is development research—that is, research concerned with the development of specific machines, or with techniques of training personnel, or with the design of organizations. In general, this kind of research is devoted to the development of specified end items, in both machine and social systems.

The second kind of research is intelligence research. This is of two broad general types. One type, research in autointelligence, is the study and synthesis of all the information bearing on the organization's own capabilities and performance. This includes studies of productivity, inventory, cost, etc. The second general type of intelligence study deals with information involving the general physical and cultural environment of the action theater; for example, the buying habits of consumers, response of consumers to sales promotion, transportation systems, natural resource distributions, etc., and also studies of the competitors involved—including all information on competitor capabilities.

The third kind of research is operations research—the comparison of existing courses of action with new proposed courses of action. Research of this type uses as raw material all information bearing on a particular set of proposed actions taken as the research problem. Existing research of this type has been done by industrial engineering and by management consultants.

Operations research is entirely concerned with research in this area.

It is important to note that most of the thinking which occurs in the organizational brain in the boxes marked "Development Research," "Intelligence Research," and "Operations Research" occurs at the subconscious level so far as the executive himself

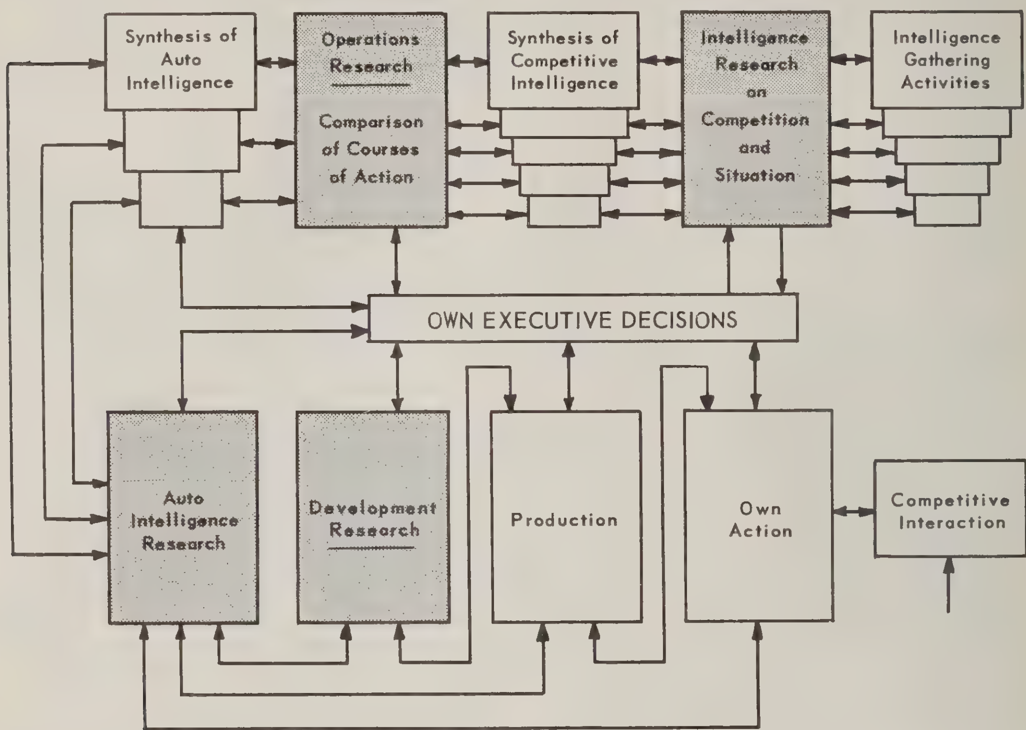

Fig. 1—Relation of Applied Research to Executive Decision

is concerned. That is to say, he sends down a question and at some later time he gets an answer; he rarely follows continuously the studies his question initiated.

It is clear from the other loops I have shown in the organizational diagram that often the executive may and does act on information directly supplied him by his action agencies.

The executive is in a position to ask questions at any level of the organization. Whenever he asks operations research a question, he must define the area and limitations of the problem that he wants to have considered. At times, however, operations research, like a part of a good and creative brain, will come up with an answer to a question that has never been asked but that has become self-evident because of the continuous correlation of

data stored in the memory banks (if the individuals doing the operations research are at all creative).

Figure 1 is primarily a diagram for the kind of operations research concerned with strategic courses of action—that is to say, with the main courses of action of a very large organization. However, operations research can occur at any level down to and including that of the individual. For large industries, a rather broad categorization can be made at three levels. Figure 2 shows the general interaction of operations research done at these three main organizational levels.

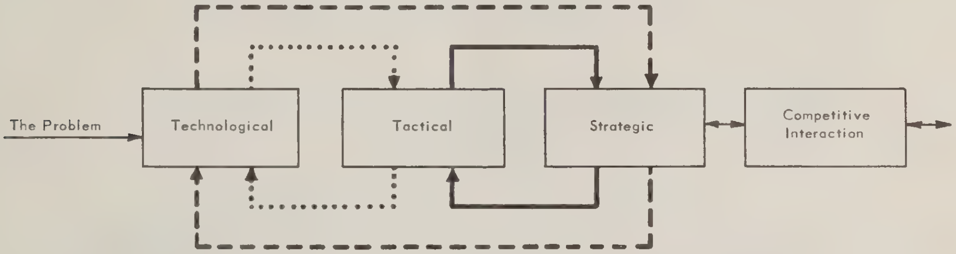

Fig. 2—Interaction between Action Levels

The lowest level is the technological level, where operations research is concerned with the comparisons of different end products—of machines, including their costs, or production line design, training procedures, etc. Usually the technological ends are derived from the tactical requirements, although, as the diagram illustrates, they may sometimes stem from strategic requirements or interaction with competitors.

At the intermediate, tactical level, the primary concern is in alternate ways of using existing or future technological means to achieve the strategic requirements of the organization. Such studies might be concerned with different kinds of sales promotion, alternative production methods in a broad sense, the location of alternative specific merchandising outlets or of factories, the manufacture of a new product, etc. The tactical ends are usually established by the strategic requirements and by the assumptions on which initiation of studies is based. As in the case of the technological studies, the tactical studies may be influenced by feedback from technological, strategic, or competitor interactions.

The top, or strategic, level for operations research is concerned primarily with the main broad courses of action of the organization. For example, should the organization operate on a stable, long-range, low-profit basis or on a high-profit, short-range, unstable market basis? Although strategic operations

research is modified by interactions with the technological, tactical, and competitor actions, in general its ends will be specified by the executive on the basis of organizational policy.

In Fig. 3 I have illustrated the interactions among operations research, the planning function, and the three main sources of

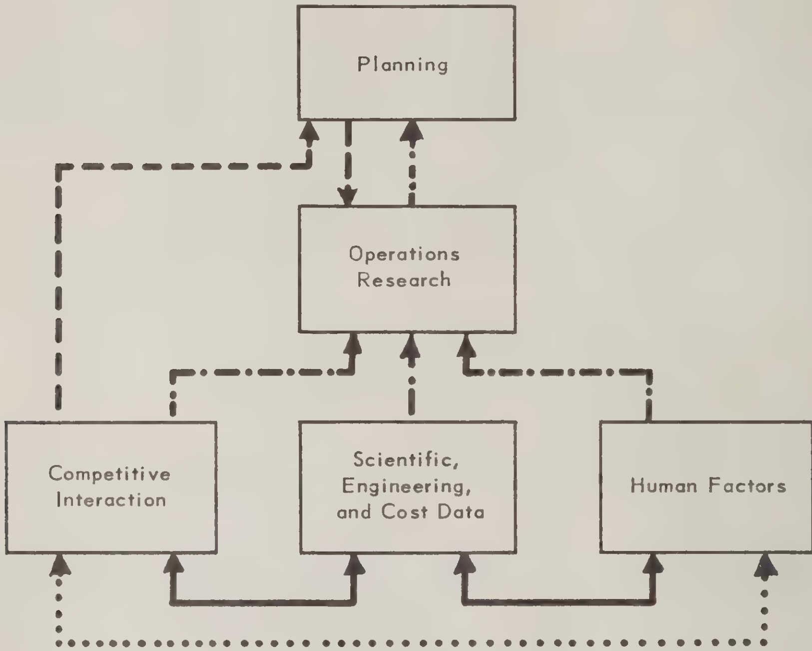

Fig. 3—Interaction between Action Parameters

data. The technological factors and the human factors involved in an operations research study come primarily from the basic, applied, or engineering sciences. The systems factors come either from passive observation and analysis of past or present operating systems—one's own or competing systems—or from deliberately designed operational experiments in which the existing organization interacts with its consumers or competitors.

II. THE OPERATIONS RESEARCH CHECKLIST FOR SOLVING ACTION PROBLEMS

I want next to introduce you to the operations research checklist. This is shown in the two following tables.

<u>Operations Research Checklist—Formulation of the Problem and Preliminary Decision</u>

1. Study of Result to be Accomplished
2. Relative Competitive Strength
3. Appraisal of the Importance of Problem
4. Preliminary Determination of Courses of Action
5. Competitor Courses of Action
6. Selection of the Best Course of Action
7. The Preliminary Decision

<u>Operations Research Checklist—Calculation of the Problem and Final Decision</u>

8. Assumptions
9. Alternative Plans
10. Determination of Operations
11. Organization of Tasks and Forces
12. Assembly of Measures for Freedom of Action
13. Assembly of Information
14. Preparation of Subsidiary Plans
15. The Final Decision

As you will easily recognize, this is a simple, brief statement of Aristotle's method for solving problems and has been adapted from the military "estimate of the situation." It is essentially a checklist for a scientific or engineering step-wise procedure for solving industrial problems and has been found very useful in operations research.

ESTIMATE OF THE COMPETITIVE SITUATION

The first of the preceding tables shows the steps that need to be taken in order to arrive at a preliminary decision. In a brief outline of these steps you will recognize the use made not only of information coming from developmental and intelligence research but from systems information as well.

ESTABLISHMENT OF THE BASIS FOR SOLUTION OF THE PROBLEM

1. Study of the Result to Be Accomplished

   A. Summary of the Situation

   This is a statement of known information which will serve as a background for visualizing the problem. Broad disposition of the organizational units, the progress of the industry, and orders received may be included. Details should not be included; these come later. Don't draw deductions. This section merely presents a broad picture of the action situation.

   B. Recognition of the Incentive

   The incentive will be one of the following:

   1. Orders from higher authority
   2. Previous decision creating new problems
   3. Demands of the situation

   C. Appreciation of the End to Be Attained

   Here, break down assigned action into a statement of the result desired. Be sure you understand the immediate purpose of the job which has been assigned to you or which you have determined for yourself. See how this fits into the general plan of your superior, either by analysis, directives, or deductions. If possible, deduce the results which he is trying to attain. Clarify in your own mind the "Chain of Objectives."

   D. Statement of the Problem

   Based on the foregoing sections, state accurately and briefly the job you have to do, and the reason why it is to be done, i.e., your "Task" and "Purpose." Normally, the Purpose would be to assist in carrying out the general plan of your superior.

   Examples:

   Task — Initiate intensive sales promotion campaign in Midwest.

   Purpose — To counter success of Company B in capturing market in this area.

Task — Redesign product A for manufacture at lower cost.
Purpose — To change present loss on this product to a profit.

2. Relative Competitive Strength

A. Means Available
   1. Products and their characteristics
   2. Distribution means and their capabilities
   3. Personnel
   4. Sales organization
   5. Logistics: materiel, personnel, and facilities

B. Characteristics of the Environment
   1. Topography
   2. Hydrography
   3. Weather
   4. Cultural habits affecting business
   5. Relative locations and distance
   6. Lines of transportation and supply; competitor traffic
   7. Facilities, location of raw materials
   8. Communications

C. Means Opposed
   1. Competitor distribution system and products: sales organization
   2. Personnel
   3. Materiel
   4. Logistics: facilities

D. Conclusions as to Relative Strength of Competitor

Here are listed deductions based on the facts collected in the preceding three subsections. A good form is to list one's own strength and weakness factors in a column parallel to those of the competitor. Note that one's own strength does not necessarily mean a corresponding competitor weakness.

3. Appraisal of the Importance of the Problem

Since every industrial endeavor involves some measure of risk, it is necessary to have an idea of the importance of the Task in order to decide whether its execution is worth while. Some tasks are so important that they require an all-out effort, regardless of consequences; others are less vital and should not be carried out if the risk is great. In this section, assess as best you can the importance of the mission, so that you will later be able to determine whether or not the Task is worth carrying out.

4. Preliminary Determination of Courses of Action

A. Re-examination of the Problem

Though you have already analyzed your objective, the study of relative competitor strength and characteristics of the environment may lead to the conclusion that your problem should be revised. This is particularly true in cases where the incentive arises from your own previous decision or from the demands of the situation. However, do not be misled into considering here how you are going to do your job; this subsection is devoted to a critical re-examination of what the job is and the further purpose to be served. End with a restatement of the mission.

B. Survey of Courses of Action

Here, list possible ways to accomplish your objective. This is to be done in broad terms, not by listing detailed operations. Make them very general, in order to cover all possibilities. For example, if the Task is "to maximize long-range profit...," possible courses of action might be:

1. To capture a moderate but stable part of the market.
2. To use sales promotion to establish long-range prestige of the trade name.
3. To eliminate dangerous competitors by underselling.

It is not necessary—it may even be undesirable—to specify in this section the type of production process, sales promotion, or product to be used. It is clearly wrong to list detailed operations. Try to visualize in very general terms all the different ways in which you can accomplish the job you've been given, and set them down. Eliminate only the fantastic ones; don't yet mentally reject any possibility, even though you doubt that it will work. Sometimes only one course will occur to you; rarely will you have more than four or five. If you think of more than a half dozen, then you are really thinking of detailed operations, not comprehensive courses of action. Start again, and set down only general plans for accomplishing the objective.

C. Testing Courses of Action

Now take each course of action in turn and subject it to the three tests listed below, using as a basis the material accumulated in subsections A, B, and C respectively of Section 1. Design physical and mathematical models of action and use analytical operations research tools to quantify effectiveness, costs, and values.

1. Suitability

a. Is the course of action appropriate to accomplishment of the objective?

b. Does it completely accomplish the objective? If not, does it assist in its accomplishment?

c. Does it accomplish the objective within the period of time necessary to achieve the purpose of the problem?

Quantitative results measuring effectiveness from the model are very important here.

2. Feasibility

a. Is it possible to carry out the operations contemplated, with the facilities available, against expected competitor opposition, and in the theater of operations?

b. How easy is the operation to carry out?

c. Does the course utilize one's own strengths and capitalize upon competitor weaknesses? This is generally given by model costing.

3. Acceptability

a. What are the consequences as to costs of the course of action?

b. Are the costs acceptable in view of the importance of the problem?

c. Are the costs acceptable in the event of failure?

The operations research theory of value is especially important here.

D. Listing Retained Courses of Action

The foregoing tests may cause rejection of one or more courses. List the ones you have retained as possibilities. You may here wish to combine two or more courses which alone are unsuitable or infeasible but which in combination meet the required tests. Here use symbolic logic.

5. <u>Competitor Courses of Action</u>

A. Survey of Competitor's Problem

1. Summary of the competitor's situation

Put yourself in the competitor's shoes. In what sort of situation does he find himself? What problem confronts him?

2. Analysis of the competitor's objective

What result does the competitor desire to bring about? You probably won't be able to deduce his exact problem, but you should have some idea of his general objective.

B. Survey of Competitor Capabilities

On the basis of the foregoing survey—the list of the competitor's means—what are the competitor capabilities which might affect your courses of action? Do not omit any possibility just because it is unlikely. <u>List here all competitor capabilities; don't restrict yourself to competitor intentions</u>.

C. Test of Competitor Capabilities

Now apply to possible competitor courses of action the same three tests you used for your own courses of action. Since your information about the competitor is meager, don't be too hasty in eliminating any capability; discard only those which clearly fail to meet the tests.

D. List of Retained Competitor Capabilities

6. Selection of the Best Course of Action

A. Comparison of Retained Courses

Execute, in theory, each of your own courses against each competitor capability. This involves breaking down the course of action into more detailed operations to see how it works out. Re-estimate the competitor's situation if necessary. Make further combinations of your own courses if it seems best. By this process you may or may not decide now to reject one or more of your courses.

B. Determination of the Best Course

The foregoing analysis should enable you here to compare the courses you have still retained. A tabulation of their advantages and disadvantages may be helpful in picking the best. If you have decided on a combination of two or more courses, be sure to test the combination for suitability, feasibility, and acceptability. Again use action models and analytical operations research techniques in comparison. Here use symbolic logic.

7. The Preliminary Decision

State your best course of action as your decision. Couple with it the purpose of the decision, which is the motivating task.

List any corollaries to the decision which your analysis has developed; that is, any subordinate deductions arrived at which limit the scope of the decision or affect the manner in which it is to be carried out.

PLANNING DETAILED OPERATIONS

Items 8 to 15 of the operations research checklist show the steps necessary in planning the detailed operations leading to a final presentation of the alternative possibilities for organizational improvement. Again, the information comes from the same sources as were discussed previously. The steps are outlined briefly in the following paragraphs.

8. Assumptions

In the "Estimate" you decided upon a basic plan. This decision was based upon the best, but probably incomplete, information available. Now determine what conditions must exist if the plan is to be successful. Don't list probabilities or expectations; only list the assumptions on which your plan is based—that is, the facts or conditions which may or may not exist but which must exist if your plan is to be successful.

For example, if the decision were "to eliminate competitor A by underselling him," a proper assumption might be: "Competitor A does not have the necessary capital to survive price competition for more than six months." On the other hand, the statement "Competitor A will meet the price competition" is not an assumption. Both statements are expressions of what opposition is anticipated; both may or may not be true, but the first one is a condition which must be true if the plan is to be successfully carried out, while the second need not be. An assumption then, as used herein, is a condition whose existence is essential to the successful execution of the plan. If, however, you are preparing an order, not a plan, then assumptions have no place. An operational plan is to be carried out only if the assumptions stated in it are true; an operational order is to be carried out (subject to the usual exceptions) without qualification.

9. Alternative Plans

It may be desirable to devise two or more plans based on different assumptions or sets of assumptions. A typical example of alternative plans would be: Plan A, based on the assumption that an inflationary economy will exist; and Plan B, based on the assumption that a deflationary economy will exist.

10. Determination of Operations

There are many ways of determining detailed operations. The one here suggested will tend to prevent omitting an essential element. Any successful industrial operation has the following constituent characteristics: effective action with relation to correct physical objectives; projection of action from advantageous relative positions; proper apportionment of competitive strength; and assurance of adequate freedom of action.

Knowing the result that you wish to obtain (the decision arrived at in the Estimate), break it down into the following elements.

A. Effective Action with Relation to Correct Physical Objectives

1. What are the correct physical objectives? These may be industrial locations; sales areas; technical personnel, ships or other transport means; development laboratories or factories.

2. What action with respect to these objectives will assist in carrying out the plan?

3. Consider the possibilities of concealing from your competitor (e. g., by feints) your ultimate physical objective.

B. Advantageous Relative Position

From what geographical locations can the contemplated action be projected? Is a change in position or a movement of facilities necessary? Here consider time and space factors.

C. Measures for Freedom of Action

1. Provisions for exercise of authority, including communications
2. Effective training
3. Security measures
4. Intelligence and counterintelligence
5. Morale
6. Plans for surprise, if desirable
7. Plans for retaining the initiative
8. Logistics support
9. Measures for cooperation

This list is not all-inclusive.

D. Apportionment of Competitive Strength

1. Consider your own strength and weakness factors. Determine whether it is better to oppose your own strength to competitor strength and dispose of his strong points first, or whether it is better to attack first where he is weak.

2. Consider strategic as well as tactical aspects.

3. Allocate adequate organizational units to each operation, from the point of view of suitability, feasibility, and acceptability.

Now is an appropriate time to test each operation for suitability, feasibility, and acceptability. Make further breakdowns or combinations; discard operations or measures which do not meet the tests; list or check those retained. Again use action models.

11. Organization of Tasks and Forces

In the foregoing section, the decision was broken down into detailed operations, and the forces available were apportioned. These detailed operations should have been stated in terms of objectives. For example, under consideration of measures for freedom of action, it might have been decided "to avoid disclosing prematurely the sales promotion campaign." Stated in terms of tasks, this becomes: "Avoid premature approach to advertising outlets." Frequently, operations can be rephrased as tasks merely by removing the preposition "to."

These tasks must be grouped according to the units which will carry them out, and the units themselves will usually have to be broken down into subunits performing the same or similar tasks. No rule of thumb is possible; too great subdivision usually complicates the allocation of authority, whereas combining too many units in one group will tend to make effective control more difficult.

Some tasks will be applicable to all units; these are grouped in one place to avoid repetition.

Complete the organization by noting the executive for each group of units.

You will now have a list of groups of units with the tasks that each performs. These must be tested, from the point of view of the subordinate executive who is to carry them out, for suitability, feasibility, and acceptability. This step is very important. It is not a repetition of previous tests, but the final analysis after the specific forces have been assigned to specific jobs.

12. Assembly of Measures for Freedom of Action

a. Measures required for security, for cooperation, and for intelligence activities.

b. Measures for logistics support. These cover provisions for procurement and replenishment of supplies, disposition and replacement of ineffective personnel, satisfactory materiel maintenance, and the like.

c. Measures for the exercise of authority. These include provision for communications, location of operating areas, and the location of the executive.

13. Assembly of Information

It is necessary to transmit to subordinates all the information required by them to do their jobs properly. Here is the appropriate place to note what items should be furnished them.

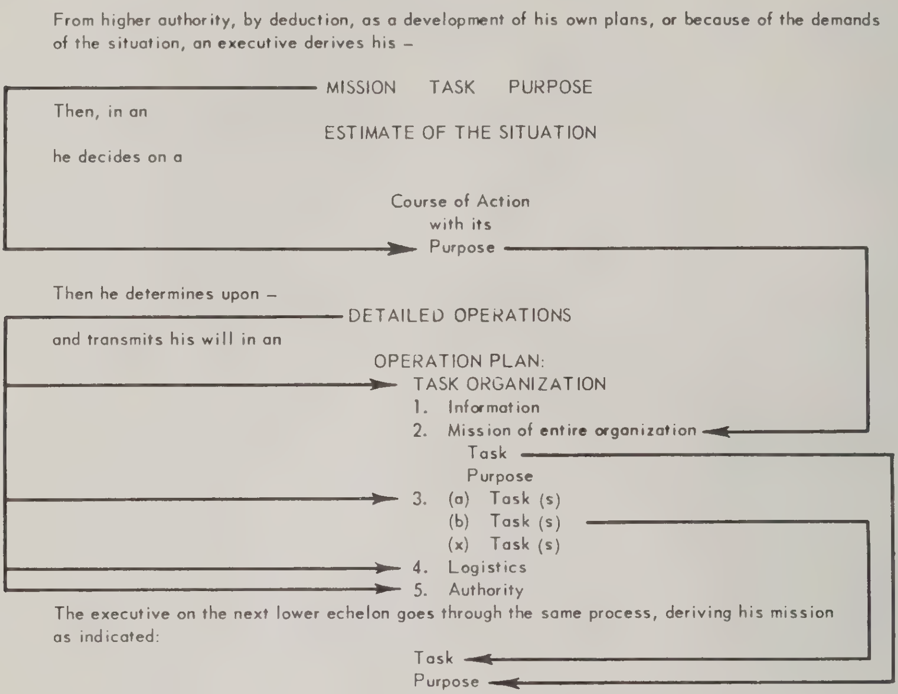

Fig. 4—The Chain of Objectives

14. Preparation of Subsidiary Plans

Either the estimate itself or the determination of detailed operations may have created subsidiary problems, for example, it may be necessary to prepare training plans, intelligence plans, or, in the field of operations proper, production schedules, detailed sales plans, etc.

These problems lend themselves to the same treatment as that given the basic plans, just discussed.

15. The Final Decision

On the basis of the preceding studies, announce your conclusions and recommendations. Include in your conclusions the estimates of effectiveness and costs of solving the problem, and the value to the company. In Fig. 4, which bears on this problem, I show diagrammatically the way in which the executive will use this kind of information.

III. SOME SELECTED ANALYTICAL TOOLS USED IN OPERATIONS RESEARCH

Having outlined the operations research checklist, I want to discuss briefly the techniques that have been found most useful in operations research. They are listed in the following table. None of these will be exactly new to you, but in operations research model-description they have been found to be especially applicable.

Some Important Analytical Tools of Opsearch

- Probability and Statistics
- Symbolic Logic
- Theory of Value
- Queuing Theory
- Stochastic Processes: The Monte Carlo Method
- Suboptimization
- Theory of Games

PROBABILITY AND STATISTICS

First of all, very broad and extensive use is made of probability and statistical analysis. Since this is so well outlined by Morse and Kimball* in their book and is such a generally used tool in industrial engineering, I will not describe the particular mutations used in operations research.

SYMBOLIC LOGIC IN OPERATIONS RESEARCH

As a preparatory step in arriving at any decision, the consequences of possible choices, as well as the relationships among known data, are examined. In certain simple cases the meanings of the facts are well-known, and the results of possible choices are predictable. When decisions involving complexly related data

*See Selected References.

are required, opsearch vision can become dimmed as a result of failure to interpret as fully as possible the many component elements in a situation. It is frequently the case that an operations analyst does not have enough facts to support an important choice; it is perhaps more frequently true that the implications of the facts which <u>are</u> available have been only imperfectly recognized.

Statements about facts can be symbolized by single letters, and separate statements can be connected by a symbol which represents the relating word between them. Such relational words are <u>and</u>, <u>or</u>, <u>if</u>, <u>then</u>, <u>neither</u>, etc. When several statements are interconnected by appropriate relational symbols, complex situations can be concisely expressed. The pattern set up among these symbols immediately discloses situations which do not fit into the pattern—cases which are inconsistent with the statements. We have a sort of symbolic picture which might easily be worth ten-thousand words.

Symbolic logic is the tool which can be used for the analysis of a pattern so designated. It may be looked upon as a sort of generalized mathematics, dealing with statements instead of numbers. It defines the manipulations and transformations which are logically permissible. Symbolic logic, then, operates to indicate what inferences can reasonably be drawn from described situations, and what inferences are false.

An example drawn from a production engineering problem will serve to illustrate one application of this tool for valid reasoning.

For some years a manufacturer had been producing a mechanism composed of three intermeshed parts. Recent researches into the properties of metals, however, had disclosed the possibility of producing a better product at less unit cost. In the case at hand, six different metals might have been used for one of the three parts, five for another, and three for the third. There was, therefore, the mathematical possibility that 90 new mechanisms would need to be compared with the old mechanism in order to be certain that all cases had been considered. Research and development efforts are expensive in both time and money, and a means was sought to reduce the number of new combinations to be investigated.

The engineering department prepared a list of specifications, reading about as follows: (1) If metal A is used in the first component, then metals D and E should not be used in the second, and metal J should not be used in the third. (Presumably these restrictions were imposed because of discovered frictional and

electrical properties which could not be tolerated.) (2) If metal B is used in component one, then metals D and E should not be used in component two, and metal H should not be used in component three. The full list of restrictions would have been the delight of a Sunday puzzle worker. A second list, describing the expected performance characteristics of the components, was also drawn up. It may be observed that 28 statements were prepared describing in full the known properties of the components. By implication, these were also known facts about the 90 possible cases under consideration.

The cost accounting department calculated the relationship between the costs of the new and old materials, and estimated the cost of converting to the new production methods. Armed with these additional estimates, and with a policy decision outlining permissible cost levels, the analyst had enough facts at his disposal to prepare a positive recommendation as to which of the 90 possible combinations should be examined further.

It is of interest to note that in this case the problem was solved by use of symbolic logic. Engineering specifications eliminated 73 of the combinations, and cost engineering specifications reduced the list by an additional 15. The remaining two combinations were, therefore, the only two cases consistent with all the demands of the manufacturer.

Once the facts were available, symbolic rules in this case could be listed, and results obtained, in less than an hour. High-speed computers could have solved the problem in several seconds. In this case, however, there were not enough complications to warrant the use of a computer, since the problem could be solved by hand in a matter of minutes.

THE THEORY OF VALUE WITH RESPECT TO INDUSTRIAL OPERATIONS RESEARCH

It is pertinent to describe in brief the development of the Theory of Value, or Worth or Importance, as you may prefer to call it, with respect to military operations research, and to make some speculation concerning its development with respect to industrial operations analysis. Decision is based on two types of consideration: consideration of probability — that is, the probability of occurrence of various possible outcomes; and consideration of the worth, or value, of these outcomes to the executive. The product of probability and value summed over all possible outcomes

is the expected value, or simply the expectation with respect to the given course of procedure.

The problem of elementary decision is to ascertain the course of action of greatest expectation; the indicated choice, of course, is this particular course of action.

We now can give a more concise and a more all-embracing definition of operations research as simply "the science of decision." What remains, then, for the executive to do? Analysis will show that the values of intermediate states in the military system depend ultimately on the probability of winning or losing the war; that is, on the value of certain states which end the fight, which we call trapped states. The absolute evaluation of these outcomes is arbitrary and must be stated axiomatically. For this reason, if for no other, the science of decision can never replace the executive, who is involved in all aspects of decision, whether probabilistic, deterministic, intuitive, or axiomatic. It should be as much the function of the operations analyst to aid the executive in the estimation of values, based upon his particular set of axioms, as in the estimation of probabilities.

All of the currently used and, at present, vaguely defined words peculiar to operations research can now be given precise operational definitions in terms of the value concept. The "Effectiveness" of a system is revealed to mean the change-in-state value brought about by the use of the system. This can be broken down into enemy value destroyed and friendly value consumed. The friendly value consumed may or may not be the cost of the system, but must include the values of the intrinsically valuable quantities consumed.

Another aspect of decision that deserves attention is the element of conflict. In decision, conflict is the rule rather than the exception. Conflict occurs whenever the individual making the decision exists simultaneously in several systems: for instance, his self, family, business, lodge, church, and nation systems. He will frequently arrive at a point where a particular course of action would increase the expectation in one of his systems and decrease it in another. Conflict as thus described frequently occurs, for instance, between the self system and the group system. How are such conflicts resolved? Several solutions are possible: (1) repression or sublimation; for instance, the self system can be devaluated, considered small with respect to the group, and thus allow decisions in favor of the group; or (2) appeal to the set of supervalues and constraints imposed by the government; that is, an appeal to law; or (3) appeal to a set of supervalues which

are not officially held and which may be written or unwritten, but are those included in the moral or ethical standards of our culture; or (4) most importantly, enlargement of the system considered in order to include all the conflicting systems as subelements. To establish a single set of scalar values in this new, enlarged system may require the establishment of new fundamental axioms which in themselves may be crystallizations of formerly accepted supervalues. For instance, in resolving the conflict between the thirteen original states, the coalition of states (the United States of America) required a set of axioms (the Constitution), the writers of which were guided by a new axiom of government (to quote from the Declaration of Independence): "We hold these truths to be self-evident, that all men are created equal..." and so on. This general method of resolution of conflict, by the enlargement of the system of attention and the readjustment of the axiomatic structure, is the fundamental operation of the rational process, whether it involves the making of decisions, the establishment of new policy, or the formulation of new physical theory.

The goal of this process is the ability to rank all states of all systems by preference; that is, to establish a scalar set of values rather than to have separate values that depend on money, morals, ethics, or religion — all mutually exclusive. One can consider the set of facts making up our body of knowledge and either simply catalog them as a separate set of unrelated points, or, preferably, establish a system of axioms on the basis of which the entire set of facts may be regenerated by direct operational means. The establishment of values is such a process, and one assumes axiomatically that this is a desirable process.

Thus a new facet to the function of operations research is revealed; in this sense, operations analysis is one of the most fundamental of all sciences, in that it is concerned not only with the content of the sciences but with their rational basis of axioms as well.

The Theory of Value can be expected to have far-reaching effects on industrial operations research (or any operations research, for that matter) in that, first, it establishes a complete rationale for that science, and, second, it gives meaningful operational definitions to its concepts.

The Theory of Value has not yet been extended to a single unified system composed of many subsystems, such as a society during peacetime. It is to be expected that operations research in industry is intrinsically more difficult than operations research in the military. In the latter, one deals with a comparatively

simple "two individual" system; that is, the values with respect to the desire to win dominate all the factional values involved. This is not true in an industrial case, nor is it true that the ultimate monistic values can be set easily. They are a function of the culture, the ethics, and the political and social theory of our nation. Thus the Theory of Value pertains to society at large and can be expected to embrace parts or all of the economic, social, cultural, anthropological, political, and managerial sciences.

Values on which industrial decisions are based are not always determined by the profit and loss motif. One must not only pay his stockholders well but must also pay his workers well, while maintaining a secure and stable company. One is constrained to operate within certain fixed conditions imposed by law, and is judged externally by values based on fundamental axioms of the rights of individuals and corporations and of the nation as a whole. The wealth of an industry is in a certain sense shared by every individual of the nation, in that all may utilize the services of that industry. A clarification of these shared values and their weight in industrial decision will be contributed, it is hoped, by the Theory of Value.

While I have not given examples of any precise application of value theory to industrial operations research, and have dealt much of the time in generalities, I have outlined a broad field of fundamental research in operations analysis from which it is expected useful results will be obtained. This expectation is reinforced by experience in military operations research, where value theory is guiding methodology in a comparison of complex and combined weapons systems, and is giving means by which to carry out operations analysis with due respect for the intrinsic values of human lives and of strategic material.

QUEUING THEORY

Queuing theory is a recent development that has application to operational systems with the following characteristics: (a) some sort of units which must be serviced, such as customers in a shop; (b) service which must be performed on them, which may or may not be independent of the number and type of customers; (c) an arrival time distribution for the "customers" — usually a random or partly periodic distribution; (d) lastly, an exit procedure by which the units leave the service point.

The theory is applicable to those cases in which a waiting line or "queue" of customers begins to form. Sometimes the queue is just what one would expect to find—a line of people waiting for bus or cafeteria service —but just as frequently no such physical line is obvious. As an example of the latter situation, a waiter's customers in a large restaurant are randomly distributed, but, from his standpoint, they form a queue (although the service sometimes seems random to us). Less frivolous examples, with obvious queues, are: (a) ore boats arriving at an unloading point; (b) parts standing on an assembly line when all has not gone well in some other section of a factory; and (c) scheduled or non-scheduled aircraft arrivals during foul weather or during large military operations. Some more subtle queues are: (a) requests for service by telephone customers or a doctor's patients; (b) transportation difficulties with unscheduled trains or overloaded facilities; and (c) payment of claims by an insurance company during a disaster.

In many cases the answers to the problems are obvious. When the demands of the queue members for service greatly exceed the capacity of the system, oversaturation is sure to occur; if the demands are well below the service capacity, no problem exists. Unfortunately, for economical reasons most systems are designed to fall between these extremes, and it is in this region, between complete saturation and underuse, that queuing theory applies. Just how delicate is the balance between these two cases may be shown by an example provided by D. V. Lindley, of Cambridge University. Suppose a service can handle exactly ten customers per hour and that an average of ten per hour arrives, but at random times. Eventually a queue will be established, subsequent to which the probability of a customer's having to wait approaches 1, and the mean waiting time becomes infinity. Further, if a slight increase in the service time is caused by the confusion arising from the length of the queue, saturation occurs at a much lower value. On the other hand, if the arrival rate is decreased to eight customers per hour, then the probability of having to wait is only 3/5, and the mean waiting time is reduced from infinity to 4.9 minutes.

Many more complicated problems occur, not all of which are presently solvable. An important example is that of aircraft approaching a taxiway from lanes around hangars. In such networks with many loops there is a positive feed-back mechanism that causes delay to be amplified, so that situations arise which, though far from producing real saturation of facilities, do result

in both multiply connected queues and disaster as far as scheduling goes. Trial-and-error techniques will sometimes give an answer to such problems, but mathematical techniques must be developed for the more important ones.

From an operations research standpoint, the answer is not, usually, to provide more equipment to avoid saturation conditions, but to study the behavior and organization of queues (this is marshalling theory) and to use available facilities to the best advantage in defeating queue formation. Because of the precarious, unstable nature of queues, small changes in the organization of facilities may result in savings in service equipment, but the even greater savings in the time of the serviced units will greatly outweigh these. Mathematically, queuing theory draws heavily on probability theory, statistics, and analysis. The presently insoluble problems may well be approached through game theory techniques, with the result that large-scale calculating machinery will be used. A large effort in these problems is justified, because it is in complex systems that queues are most likely to appear and are most disastrous; in such systems their avoidance would result in the largest payoff.

STOCHASTIC PROCESSES: THE MONTE CARLO METHOD

It is not surprising that many of the powerful mathematical methods of the physical sciences can be used in operations research, when one considers the similarity of the problem-generating systems. An example is the Monte Carlo method, which has become increasingly important since the advent of large-scale calculating machinery. As the name implies, the method makes use of large tables of random numbers, of the type that certain gaming devices are supposed to generate. This is just one of a group of calculational techniques called "stochastic" processes because of their use of random numbers. The term "Monte Carlo," however, is sometimes used to describe the whole class.

The Monte Carlo method itself consists of substituting a stochastic (or probabilistic) procedure for another mathematical (usually analytic) model of the system. The merit of the method lies in its applicability to those cases which require a numerical answer difficult to obtain analytically. The usual problem is to find a stochastic process that has a distribution function (or the probabilities involved in the choices) which corresponds to the physical problem. Once the process is found it should (and usually

does) consist of a set of very simple calculations that must be performed many times. For instance, one can consider a mobile particle in a field of stationary ones. The particle takes a velocity and direction determined by a random number and eventually meets another particle. It may be captured or have an elastic collision in accordance with certain fixed probabilities. Its fate is again determined by a random number. This is an ideal situation for a large calculator, with which a large number of runs can be made quickly. Probability theory, in the form of the law of large numbers, tells us that the physical result is approached more and more closely as more runs are made.

Examples of simple yet valuable Monte Carlo processes are not hard to find. One such is used in the ruin problem of a gambler, which has an analogue in atomic pile theory. The analytical formulation is not difficult, but numerical results are not easily obtainable from it. It is considerably simpler and less expensive to play the actual game by machine many times to obtain the distribution of ruin plays. Many physical problems can be expressed as games in which repeated play is the best recourse for finding the outcome.

An important quality of the method is that a stochastic formulation is frequently very nearly like the physical situation. To be more specific, problems that involve numbers of individual particles, such as showers in electron multipliers, or neutron ages in media, are frequently formulated in terms of continuous variables. By the Monte Carlo approach, however, a representative sample of particles is followed through its history and the final distribution found. Some variables which are hidden by their implicit appearance in the continuous case are easily found in the stochastic process, and their effects studied.

Monte Carlo techniques are not limited to problems of the game type. The tedious but common process of the solution of simultaneous equations can be approached by this method, as can the partial differential equations of heat flow and electromagnetic theory. Fortunately, the actual calculation in most cases is quite simple; the use of the automatic calculator is important because of the great bulk of the work.

The Monte Carlo technique appears to be applicable to problems in sales campaigns, traffic handling at seaports and airports, communication, scheduling, and the design of sales and production experiments.

THEORY OF GAMES

The Theory of Games is a relatively new branch of applied mathematics. It deals with situations in which each of several people can partially influence the outcome of a certain event, but no one of them alone can determine the outcome completely. Each acts so as to influence the outcome according to his preference.

The theory was originally developed to provide a mathematical basis for economics. More recently it has been applied to problems in sociology and in military tactics.

A considerable body of theory has been developed for zero-sum, two-person games — zero-sum in the sense that no wealth is created or destroyed, but only transferred between the contestants. One of the simplest games of this type is the game of matching pennies. The whole point of this game is to outguess one's adversary. Neither player can adopt a fixed pattern of calling heads or tails, for if he did, his opponent could anticipate this pattern and call his coins accordingly. However, by calling his coins at random, either player can insure that he will not suffer a net loss in the long run. Thus the game is solved in the sense that an optimum strategy is given for each player, and the expected outcome is determined if each player employs his optimum strategy.

A central theorem in the Theory of Games is Von Neumann's "minimax theorem." This theorem states that every finite zero-sum, two-person game has optimum strategies for the two players, and a unique expected outcome for these strategies.

Optimum strategies have been worked out for quite a number of particular games. Also, several general procedures have been devised which will theoretically yield the optimum strategies for any finite zero-sum, two-person game. However, for a game with many choices, the amount of work involved in applying these procedures is prohibitive. Chess is an example of such a game.

The theory of non-zero-sum games is not in nearly so satisfactory a state. Yet it is very important, since it applies to economic processes in which wealth is created and to military processes in which wealth is destroyed.

Also, there is as yet no well-established theory for multiple-person games, although studies have been made of the conditions under which alliances and coalitions are formed.

The theory of zero-sum, two-person games can at present be useful to the operations analyst. The theory of more general types of games needs further development.

Before leaving the problem of techniques, I want to illustrate (see Fig. 5) and discuss the three main regions in which most models appear to fall. Although usually constructed for only one region, a model may at times encompass all three regions.

The first is the attrition or linear region, in which linear models may be very readily employed. This, of course, is the

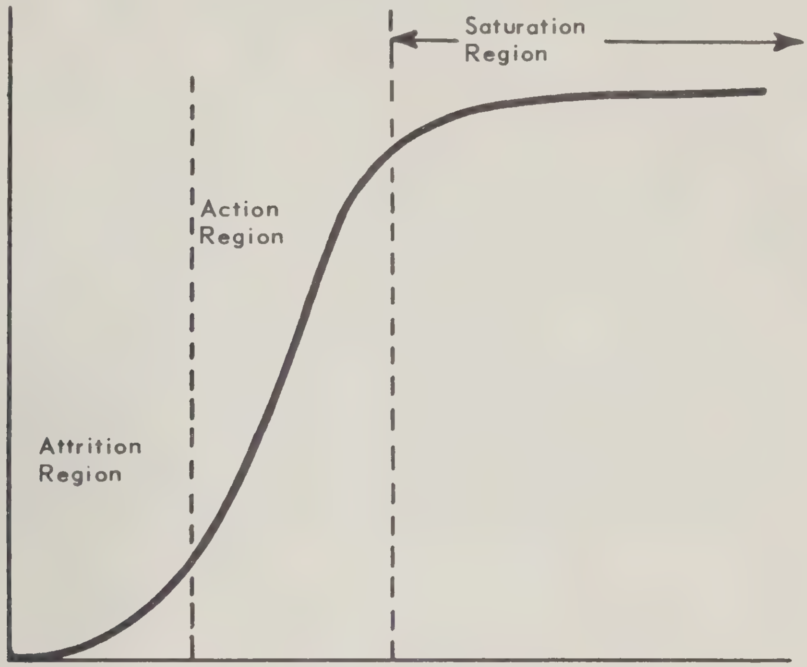

Fig. 5—The Three Areas of Opsearch

easiest region in which to construct models. The second is the action region, which, although nonlinear, may sometimes be approximated by a linear model. The third is the saturation region, which is completely nonlinear, and is the most difficult to handle.

A simple wartime example, illustrating the importance of these three regions, is found in the situation of ship repair in Japan as a result of submarine and mine attacks. Japan had limited repair facilities, and when the number of damaged Japanese ships was small, damage to ships had only small detrimental effect on shipping because of the high repair capacity, and counted for little. This was in the attrition region. When the damage began to be much more extensive, in the action region, the effect on Japanese shipping capabilities became marked, as queues formed waiting for repair. In this region a damaged ship went

from a value of 0.1 to 0.4 of a kill. With a high increase in the number of damaged ships and with repair facilities completely saturated, very long waiting queues formed, and a damaged ship became the equivalent of a killed ship.

Another example is the effect of fatigue and fear on the individual in combat, where the human factor must be considered as a nonlinear problem. It is well known that there is a relation between fatigue and fear. Fatigue can be caused not only by physical work but by emotional work. Thus, a man who is afraid will become fatigued, and as he becomes more fatigued he becomes afraid more easily. This can become cumulative and disastrous. If the battle is light and the individual is in good physical condition, he can endure battle for a long time. As his physical exertion and the danger in battle becomes greater, his endurance decreases sharply in the action region. A most exhausting situation in the saturation region occurred in the landings at Normandy, where the waiting was long under very heavy and determined German attack. The fear engendered by the situation was so intense that it brought on extreme physical fatigue, so great that many men drowned in a surf that ordinarily would not have been at all dangerous. There are many other examples of social behavior of this type which occur in industry.

IV. OPERATIONS RESEARCH CASE HISTORIES

SEA MINING AGAINST JAPAN

Next, as illustrations, I want to give you the results of several operations research studies. The first one is taken from my own experience in the Pacific in the use of sea mines against Japanese shipping. This particular campaign was conducted entirely on the basis of findings derived in operations research studies.

The first studies were made at the technological level at the Ordnance Laboratory in the Washington Navy Yard. They had been preceded by primitive strategic and tactical opsearch studies. The weapon studies were concerned with the comparison of simple and combination influence-mine mechanisms, contrasting their ability to withstand enemy mine sweeping and their effectiveness in sinking enemy ships, at various depths of enemy waters. For example, the target widths and sweep-proofness of magnetic and pressure influence mines were compared. Such problems as the need for flexibility in meeting enemy technological improvements during the course of the mining campaign were also studied. Results of these analyses, as an aid to developmental research, were seen in a family of very effective aircraft-laid sea mines.

The second phase of the operations research was at the tactical level. Typical was the analysis made at the 21st Bomber Command at Guam. At that time the bomber command was flying its B-29s in daylight at 20,000 feet, in formation, using visual bombing. The attrition rate against the aircraft was exceedingly high, ranging up to 10 and 15 percent. This was actually in excess of the sustained combat abilities of the aircraft crews.

An analysis was made which showed that in addition to enemy fighter attack, one of the main causes of the heavy losses was the wearing out of engines because of overload resulting from flying to the high altitudes specified and flying in formation. This resulted in many operational ditchings over Japan and on the return trip. The basic heavy losses due to mechanical wear were

increased because the weather was good only infrequently and then only for a period of days. During good weather, because of continuously scheduled operations, maintenance crews were overloaded to the point where they were unable to do good work because of excessive fatigue.

After consideration of all the facts bearing on the problem, a preliminary decision was made to consider a course of action in which the aircraft operated in single flights, at night, using radar bombing, at an altitute of about 5000 feet. An analysis of the Japanese antiaircraft defenses then showed that there was a hole in the defenses at the 5000-foot altitude, and that cloudiness, normally heavy and unfavorable for visual bombing, would prevent the Japanese use of searchlights at night. As it was believed that the Japanese had no adequate radar fire control, it was expected that their antiaircraft fire would be ineffective. Also, the Japanese did not appear to have night fighters, and losses due to interception could be expected to be negligible.

It was shown that the use of single flights at night would result in a very much lower rate of engine wear than previously, and that very adequate maintenance could be provided if the flights were made on a regular every-other-day basis, which would be possible if radar bombing were used. It was estimated that the cost of such tactics would be a loss of less than 1 percent as compared to the existing 10 to 15 percent. This prognosis was completely confirmed after the plan was adopted. Finally, it was shown that because of the ballistics of the mines, mining from low altitudes by radar would be just as accurate as visual mining from very high altitudes.

An over-all improved effectiveness of 20 to 30 times was predicted for the operations at the low altitude. On this basis the tactical plan was approved by General LeMay and was successfully carried out.

The strategic part of this research study considered the over-all shipping situation in Japan. It was shown that shipping within the enclosed Japanese Sea and the enclosed Inland Sea, with access through the Shimonoseki Strait, would be adequate to maintain Japan in the war indefinitely, and that this shipping was essentially invulnerable to submarine and plane attack. Although United States Navy submarines had reduced the Japanese shipping to quite a low level, this level was not as yet low enough to endanger the basic defense of Japan in the beginning of 1945. It was then shown by the analysis that by the use of mines in attacks on the Shimonoseki Strait and the Japanese harbors, the entire

shipping of Japan could be sunk and incapacitated by a relatively small aircraft mining effort, which might prevent Japan from continuing the war. It was on this basis that General LeMay agreed to the use of one bomber group for mining soon after he took command of the 21st Bomber Command. The predicted reduction of the shipping occurred and was quoted by the Japanese as one of the important factors leading to the surrender of Japan.

In comparing the results of the sea mining, it was shown that the logistic and military effort was approximately one-tenth as great for a mine attack as for a similar attack by submarine or direct air.

The next four examples are taken from recently reported industrial operations research studies. The first three are published in full in the February 1953 issue of the Journal of the Operations Research Society of America, Volume 1, Number 2.

THE RELIABILITY OF AIRBORNE RADAR EQUIPMENT*

This is a study of the failure rates of radar sets. This study is now used in the determination of the operational suitability of existing and future electronic mechanisms in aircraft. It is at the technological level.

Boodman's study was made to provide a guide for development research in the design of more reliable equipment of the type using many electronic and electrical components.

He found that in an equipment having $l_z + t$ components of age z, the chance that a component of age z would continue to operate to time t is

$$ {}_tP_z = \frac{l_z + t}{l_z} = e^{-n\mu T} $$

where μ is the average chance failure rate. The comparison of this equation with the failure rate of one type of aircraft radio tube is shown in Fig. 6. Further comparisons with the equation of the fraction of radar sets surviving a time t of operation are given in Figs. 7, 8, and 9.

*Condensed from David M. Boodman (OEG, Navy Dept), "The Reliability of Airborne Radar Equipment," *Journal of the Operations Research Society of America*, Vol. 1, No. 2, February 1953.

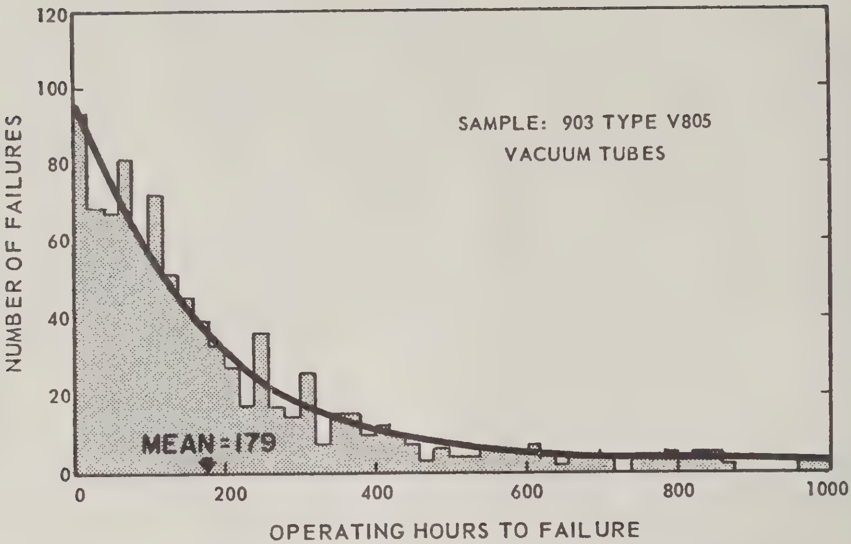

Fig. 6—Distribution of Failure of Aircraft Vacuum Tubes

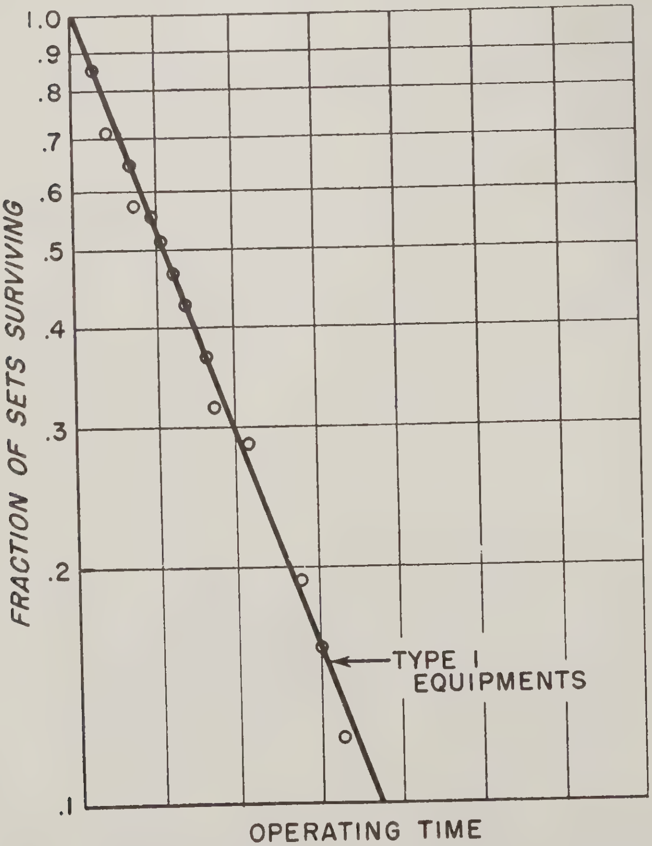

Fig. 7—Failure in Type 1 Airborne Radar Equipments

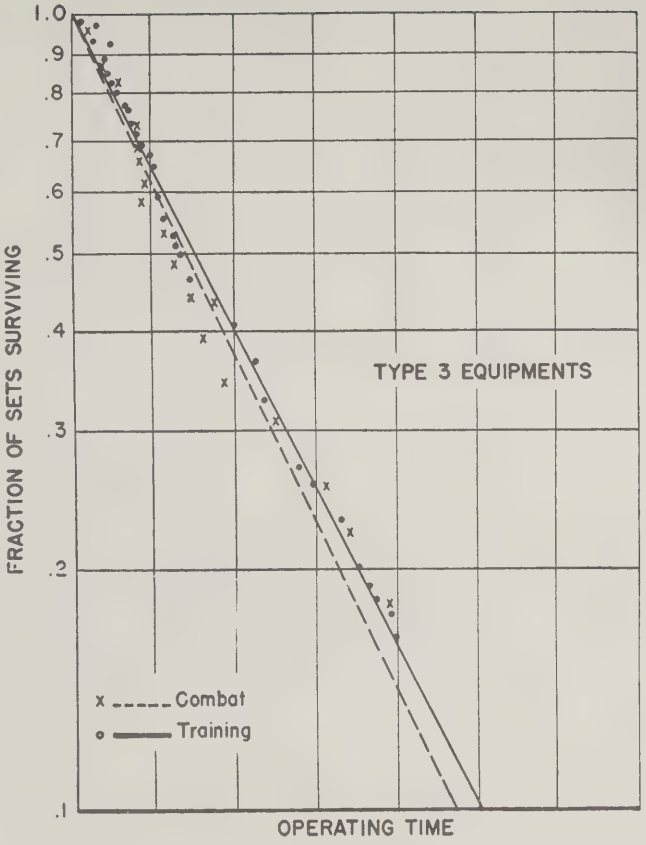

Fig. 8—Failure in Type 2 Airborne Radar Equipments

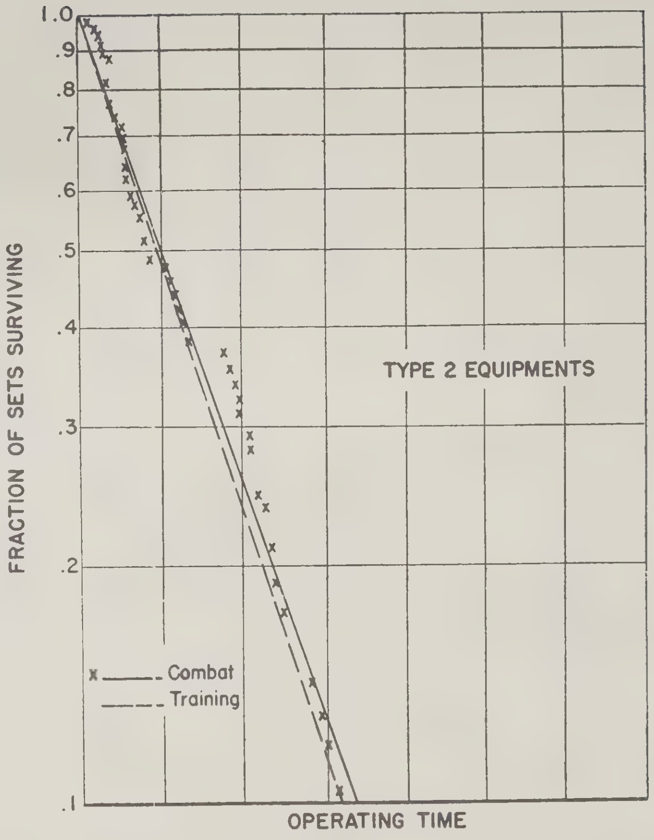

Fig. 9—Failure in Type 3 Airborne Radar Equipments

The value of μ of the sets was taken as $\mu = \mu_v V \mu_c C$, where μ_v and μ_c, V, and C are respectively the failure rates and numbers of vacuum tubes and other components vital to proper operation of the set.

The average failure rates were:

$$\mu_v = 0.0007\ hr^{-1}$$

$$\mu_c = 0.0002\ hr^{-1}.$$

There was thus found a useful life of 1400 hours for vacuum tubes and 5000 hours for other electrical parts, a small fraction of their bench life.

On this basis an apparatus with 250 tubes and 2500 other parts has a 50 percent chance of surviving 2 hours. This is exceedingly useful information in developmental research.

OPERATIONS RESEARCH IN AGRICULTURE*

Seabrook Farms is a large farming enterprise in southern New Jersey. It is a completely integrated industry which raises vegetables, processes them, quick-freezes them, stores them, and distributes them to housewives all through the eastern half of the United States. This entire operation, from farm to table in one production line, provides opportunities for a great deal of operations research.

One problem presented to Dr. Thornthwaite was that of predicting when peas would mature. This information was desired so that the peak labor force could be provided to handle the simultaneous ripening of crops in many fields. It was also desired to harvest the peas when they were just ready and of highest quality. At the time that the problem was presented, the division managers of the farm tried to harvest at a rapid rate during the period when peas ripened rapidly. If the work load could not be handled with a 12-hour day, provisions were made for a 24-hour operation, using floodlights to illuminate the fields, and double harvesting crews. This solution was not adequate; not only did queues form in harvesting, but equally important queues formed in the pile-up of the product in the factory. The freezing capacity was overtaxed, and eventually the harvest fell behind, so that some peas were overmature and of poor quality when harvested.

*Condensed from C. W. Thornthwaite, "Operations Research in Agriculture," *Journal of the Operations Research Society of America*, Vol. 1, No. 2, February 1953.

The story of the study is a long and interesting one. In his final solution, Dr. Thornthwaite developed a climatic calendar based upon the rate of growth of pea plants as a function of calendar time. This climatic calendar was found to apply to most other plants as well. The curve he derived is shown in Fig. 10. At Seabrook he found that there are 7500 growth units in a year.

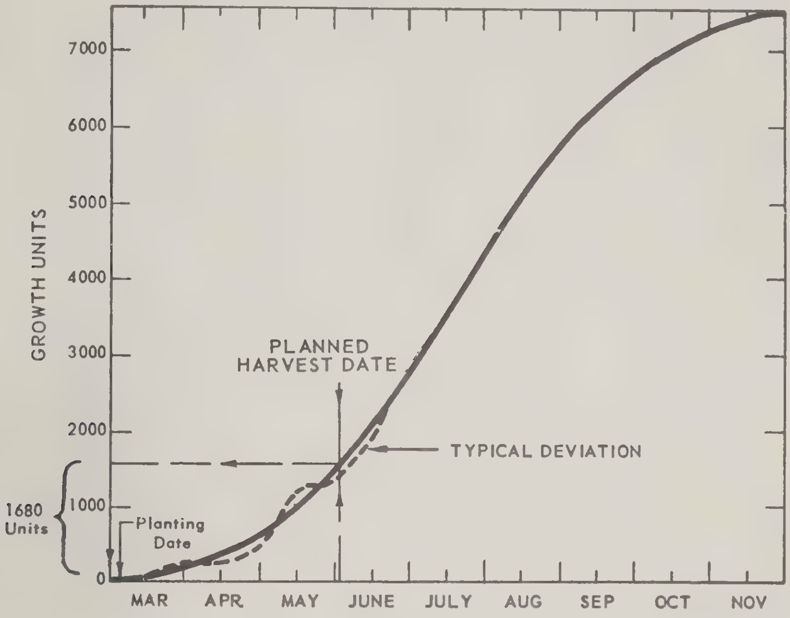

Fig. 10—Crop Growth Curve at Seabrook, N. J.

In comparison, Maine was found to have 4500 growth units. Pensacola, Florida, 12,000 growth units. These growth units are a measure of the amount and rate of growth which can take place within any period of time. Second, he found that an accurate growth index could be assigned to each variety of pea (and indeed to any other vegetable). Now, such facts as these require a great amount of experimental data before the values used in scheduling can be firmly established. For preliminary decision, made on the basis of these experimental data and data on all other operations at Seabrook, Thornthwaite proposed to reschedule planting according to the climatic calendar, in such a way that a very even rate of maturing would occur day by day just within the capacity of the harvesting, freezing, and storing facilities. He proposed to eliminate the queues by eliminating randomness in ripening. A stabilized labor force was proposed just adequate to meet the average labor need, peak labor being met by other members of the farm families. By using the climatic growth

curve, and the growth indices of the various pea varieties, which had a range of about 25 percent, it was possible to schedule pea planting so that at harvest time peas became ripe at a steady and uniform rate, with only small variations due to minor perturbations caused by variations of the climate from the average values. Contingencies due to poor weather, holidays, etc., were easily provided for.

This same scheduling was put into effect in 1950 for everything that was planted at Seabrook Farms. All planting was done according to the climatic calendar. There are still some problems left in this system, which schedules planting in the same way that railroad trains are scheduled. There are still variations due to abnormally cold or hot weather or to abnormally wet or dry weather. Fortunately these produce only minor peak loads, since the entire harvest is slowed in the same sequence.

As a result of this new course of action, labor costs for harvesting were greatly reduced. Equipment costs were much reduced by elimination of excess plant capacity, excess canning capacity, and elimination of equipment such as floodlights, and other items for night operations. The entire farming operation, including labor, was stabilized as a result of the vegetable dispatching system. It is of interest to note the simple and inexpensive nature of the solution.

It is worth pointing out that such excellent results could be achieved easily only in an integrated operation of the Seabrook type.

This is only one of a number of very important operations research studies made by Thornthwaite.

THE EFFECT OF PROMOTIONAL EFFORT ON SALES*

This is a tactical study concerned with the effectiveness of sales promotions in increasing sales. The company that was studied distributes coffee to a large number of retail grocery stores throughout the country. The promotional effort is conducted by salesmen who distribute promotional material at the point of sale. The cost of the promotion is high. The company had made studies during the last two decades to determine the best balance between sales volume and an economical amount of promotion. None of these studies led to any clear and definite conclusions.

*Condensed from John F. Magee (Arthur D. Little, Inc.), "The Effect of Promotional Effort on Sales," *Journal of the Operations Research Society of America*, Vol. 1, No. 2, February 1953.

The questions asked of the operations analyst were:

1. How good is the existing procedure for the selection of dealers whose sales should be promoted?

2. How much further effort is warranted in refining the dealer-selection method?

3. How much promotional effort is justified as long as the present method of directing this effort is retained?

Fortunately, in studying and answering these questions there was ready access to the purchase records of many thousands of dealers, and to the previous results of experiments designed to answer the company's problems.

In the model adopted, dealers were ranked from very good to very poor on the basis of the number of cases of coffee ordered per month. Data were available on the effect of sales promotion on fractions of the dealers ranging from the top 30 percent to 100 percent.

It was found that with sales promotion given to all dealers the fraction of dealers $f(n)$ ordering n cases per month was given by

$$f(n) = \frac{s^n}{(s+1)^{n+1}} \tag{1}$$

where s is the average number of cases per dealer averaged over all dealers. The fit of the equation to the data is shown in Fig. 11.

It was then shown that when the top fraction of the dealers was given sales promotion, the fraction ordering n cases was

$$f_{np}(n) = \frac{s^n}{(s+g+1)^{n+1}} \tag{2}$$

where $g = (a/1-a)$ and a is the fraction of top dealers promoted. The fit to the observed values is given in Fig. 12.

The relative efficiency due to varying amounts of promotion is shown in Fig. 13. This is the percentage increase due to selection of the percentage a as compared to a random sample selected for promotion.

Finally, it was found that profits due to promotion were optimized when

$$a = 1 - \left(\frac{1}{N_{sv}}\right)\left[\frac{d}{da}\{C(B) + C(a)\} + i\frac{d}{da}\{I(B) + I(a)\}\right] \tag{3}$$

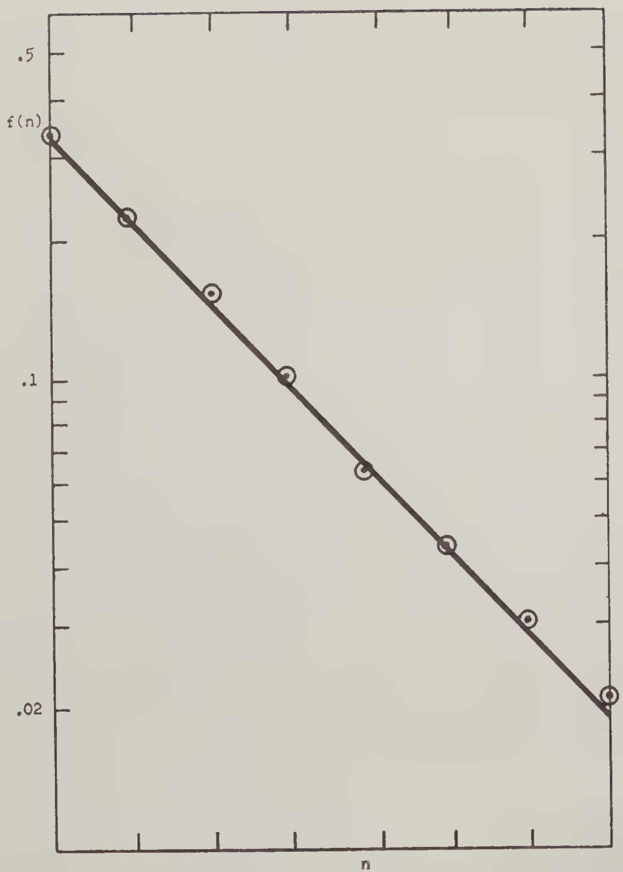

Fig. 11—The observed fractions of dealers ordering n cases in a month: all dealers given special promotional help. The full line is a plot of the equation using $s = 2$.

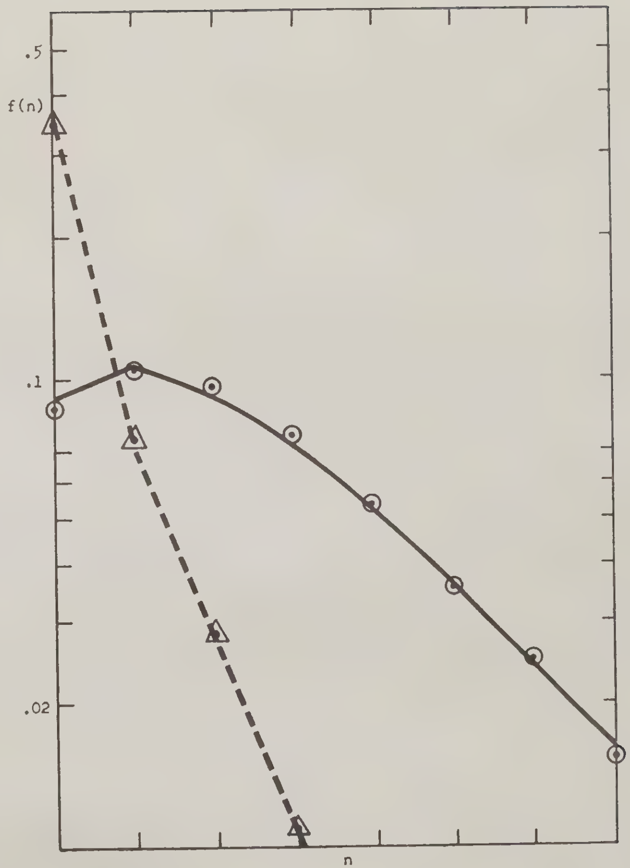

Fig. 12—The observed fractions of dealers ordering n cases in a month. Those normally receiving promotional help are plotted as circles, and those normally not receiving promotional help as triangles. The full and dotted lines are plotted from appropriate equations.

where N = the number of dealers
v = the value per case of coffee
$C(B)$ = the cost of producing and distributing a volume B of product
$C(a)$ = the cost of promoting a fraction a of dealers
i = the interest desired from investment
$I(B)$ = capital required to support production B
$I(a)$ = capital required to support promotion a

This theoretical analysis provided measures very useful in designing company tactics and strategy for not only sales promotion but also general company operations.

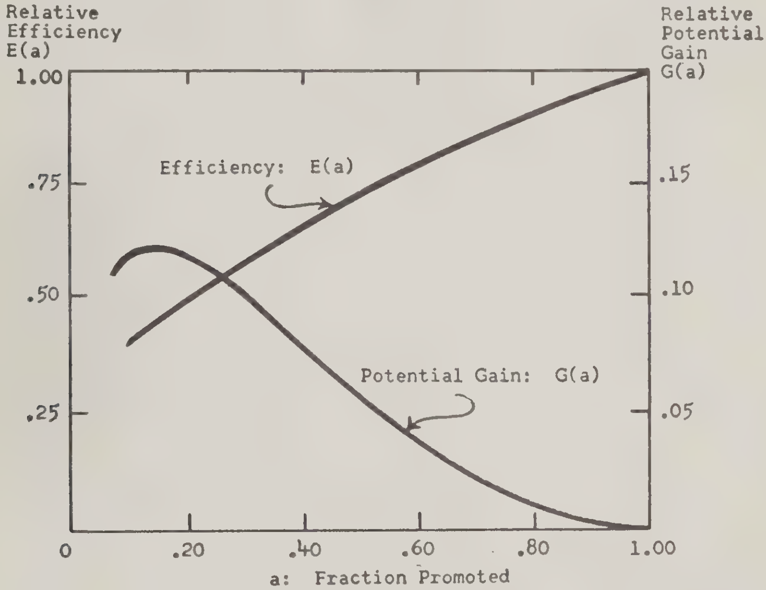

Fig. 13—Relative efficiency of distribution of promotion and potential gain in sales.

STRATEGIC OPSEARCH IN PUERTO RICO

The problem was: "How can Puerto Rico, which has virtually no natural resources, and which has a birth rate double that of the mainland United States, provide for its people a standard of living comparable to that of the mainland United States?"

The Puerto Rico Development Company called in Arthur D. Little, Inc., as consultants. The insular government, through its Development Company and with aid from Arthur D. Little, Inc., then examined all phases of the problem—psychological, social, and economic.

Their combined activities resulted in a carefully thought out program for establishment of new industries which would give Puerto Rico a broader and more stabilized economy. This program helped to introduce such industries technologically, developed and proposed programs for tax problems growing out of introduction of such industries, and conducted market analyses and market studies to help sell the island's products.

The results of these studies, which were aimed at a whole system and not at any individual problems, were put into effect, and the results speak for themselves.

The Puerto Rican rum promotion is an example of one detailed operation in the over-all approach to the island's problems. The initial studies were made by A. D. Little,Inc., on behalf of the Development Company. With tax revenues from rum reduced from a wartime peak of some $65,000,000 annually to approximately $4,000,000 in the first postwar year, it was recommended and decided that an advertising and sales promotion program to increase the market for Puerto Rican rum in the United States would be undertaken. This one activity, in which importers in the United States cooperated, resulted in doubling rum shipments to the United States in 1949, the first year of its operation.

The effect of the plan as executed led, in six years, to the following improvements in Puerto Rico's position:

Puerto Rico's world commerce up 250 percent.

Income tax payments by individuals in Puerto Rico up 400 percent.

Property evaluation increased only by 30 percent, indicating a healthy but noninflationary economy.

Puerto Rico's public debt cut in half.

School enrollment up 70 percent.

College enrollment doubled.

The island electric power production more than doubled.

In comparison with surrounding Latin American countries the indices showed:

	Puerto Rico	Mexico	Argentina
Kilowatt hours per person	200	121	190
Foreign trade	216	42	110
Literacy	68%	55%	85%

On the whole, these indices are quite favorable to Puerto Rico, in view of its much poorer situation with respect to natural resources.

DIFFICULTIES MET IN OPERATIONS RESEARCH

I think that I have made it clear that one of the primary problems of the operations analyst is the translation into executive language of conclusions and recommendations resulting from a particular methodology. Having translated his results into good common-sense executive language, the analyst has the responsibility of selling his particular conclusions and recommendations to the decision-making system, including the executive, and explaining the advantages together with the disadvantages. I have found this to be exceedingly important. I have never found it adequate simply to place a report on an executive's desk with the implication "Here it is, take it or leave it." An operations research study becomes effective in proportion to the amount of effort spent in communicating the effects of the research to the whole system as well as the executive, and clearing up with the executive on a personal basis all of the many questions involving the validity of the study. This is especially important in large organizations where the executive is subject to pressures from many subempires within his organizations. He has particular pressures from his sales and operating organizations, from his production organizations, and even from his research and development organizations, which are usually reluctant to undertake any drastically new course of action desired by management (even though this may be highly profitable in the long run to the over-all organization). Very few analysts are adept at, or recognize the need for, such multilingual ability on their part. Consequently, the results of much good operations research are never used.

I want to point out frankly that operations research has many other severe limitations. It has still to prove itself and achieve recognition. It can do so only on the basis of performance.

I regard the following factors as the most severe limitations in industrial operations research:

1. The lack of basic information on human behavior.
2. The usual lack of adequate information on competitor intentions and capabilities.

3. The extreme difficulty in getting highly skilled specialists from very diverse and often antagonistic disciplines to work well as a closely integrated team.

In the Operations Research Office it is this last factor which receives our greatest attention, i. e., attempts to form smooth-running teams composed of professionals coming from a variety of physical, social, and engineering sciences, as well as from operations.

Operations research as been vigorously opposed by many economists and econometricians. This is a curious attitude, and one which I do not pretend to understand. On the other hand, many industrial engineers and management consultants have adopted operations research as a technique that they could learn to use as a tool in their own professions.

I, myself, believe that each of the three disciplines, i.e., industrial engineering, operations research, and management consulting, serves particular aspects of the executive's need for help in designing new courses of action. I believe the three are mutually supporting. Each will benefit if all are professionally connected through affiliations of their respective societies and the establishment of mutually acceptable standards. I hope this can occur soon.

In summary, it is evident that I have been describing a method of predicting the practical outcome of an industrial operation by constructing a scientific model which accurately describes the operation and permits prediction of future actions.

A definition of operations research is then:

"Operations research is the prediction and comparison of the values, effectiveness, and costs of a set of proposed specific courses of action involving man-machine systems, and is based on a model of the action which has been analytically described by a logical and, when feasible, a mathematical methodology and which has had the values of the basic action parameters determined either from a historical analysis of past actions or from designed operations experiments. Most importantly, because all human and machine factors are meant to be included, an estimate of the uncertainty in the predicted outcome, and in the values, effectiveness, and costs of the proposed actions is provided."

The methods of operations research are straightforward. They can be adopted and used by anyone who employs common sense supported by some technical training. It is hoped that operations research will be widely adopted by industry.

APPENDIX

OPERATIONS RESEARCH ORGANIZATIONS

Firms employed by industry to do operations research:

Alderson and Sessions — Philadelphia, Pa.
Armour Research Foundation, Illinois Institute of Technology — Chicago, Ill.
Arthur D. Little, Inc. — Cambridge, Mass.
Booz, Allen & Hamilton — Chicago, Ill.
Davee, Koehnlein and Keating — Chicago, Ill.
Stanford Research Institute — Palo Alto, Calif.

Firms having their own opsearch groups:

Atlantic Refining Co. — Philadelphia, Pa.
Bell Telephone Laboratories — Whippany, N. J.
Curtis Publishing Co. — Philadelphia, Pa.
E. D. Smith and Co. — Silver Springs, Md.
Giant Food Stores — Washington, D. C.
Melpar, Inc., Westinghouse Air Brake Co. — Alexandria, Va.
Metropolitan Life Insurance Co., Actuarial Division — New York, N. Y.
Seabrook Farms, Inc. — Seabrook Farms, N. J.
Sun Oil Co. — Philadelphia, Pa.
U.S. Rubber Co. — Detroit, Mich.
U.S. Time Corp. — Waterbury, Conn.

Firms doing opsearch for military establishments:

American Institute for Research — Pittsburgh, Pa.
American Power Jet Co. — Ridgefield, N. J.
Baird Associates — Cambridge, Mass.
Battelle Memorial Institute — Columbus, Ohio
Beers and Heroy — Dallas, Tex.
Booz, Allen & Hamilton — Chicago, Ill.

Broadview Research and Development — Burlingame, Calif.
Columbia Research and Development Corp. — Columbus, Ohio
Cooperative Research Foundation — San Mateo, Calif.
Curtiss-Wright Corp. — Columbus, Ohio
Delta Research and Development Corp. — Rouston, La.
Dunlap and Associates — Stamford, Conn.
Emhart Manufacturing Co. — Hartford, Conn.
Experimental Towing Tank, Stevens Institute of Technology — Hoboken, N. J.
Haller, Raymond and Brown — State College, Pa.
International Public Opinion Research, Inc. — New York, N. Y.
Midwest Research Institute — Kansas City, Mo.
Smith and Davis, Inc. — Silver Spring, Md.
Snow and Schule, Inc. — Cambridge, Mass.
Stanford Research Institute — Palo Alto, Calif.
Technical Operations, Inc. — Arlington, Mass.

Military opsearch organizations:

Air Force—Operations Analysis Division, the Directorate of Operations, Hq. USAF — Washington, D. C.
Air Force—RAND Corp.— Santa Monica, Calif., and Washington, D. C.
Army—Operations Research Office — Chevy Chase, Md.
Joint Chiefs of Staff—Weapons Systems Evaluation Group— Washington, D. C.
Navy—Operations Evaluation Group —Washington, D. C.
Army—Operations Research Group, Chemical Corps — Edgewood, Md.

British research associations doing opsearch:

The British Boot, Shoe and Allied Trades Research Association — Kettering
British Electricity Authority — London
British Iron and Steel Research Association Institute — London
National Coal Board — Cheltenham, Gloucester
Road Research Laboratory, Department of Scientific and Industrial Research — Harmondsworth, Middlesex
The Shirley Institute (The British Cotton Industry Research Association) — Didsbury, Manchester

Seminar courses offered in opsearch:

Case Institute of Technology
Columbia University
The Johns Hopkins University
Massachusetts Institute of Technology
Pennsylvania State College
University of Illinois

The following list comes from the Operations Research Committee of the National Research Council with the remark that these firms are "strongly interested in establishing an Operations Research program":

The Baltimore and Ohio Railroad Co. — Baltimore, Md.
Chrysler Corporation — Detroit, Mich.
The Chesapeake and Ohio Railway Co. — Cleveland, Ohio.
Commercial Solvents Corporation — Terre Haute, Ind.
Eastman Kodak Co. — Rochester, N. Y.
General Electric Company — Richland, Wash.
Minnesota Mining and Manufacturing Co. — St. Paul, Minn.
Pillsbury Mills, Inc. — Minneapolis, Minn.
Republic Steel Corporation — Cleveland, Ohio
The Royal Swedish Academy of Engineering Sciences — New York, N. Y.
Schenley Industries, Inc. — New York, N. Y.
South African Council for Scientific and Industrial Research — Pretoria, S. A.
Standard Oil Company — New York, N. Y.
United Fruit Company — Boston, Mass.

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